The Effects of Training on Pre-Algebraic Pattern Thinking in Preschoolers

Undergraduate Research Thesis

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By

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ABSTRACT

A human's ability to recognize patterns in early development is predictive of later mathematic aptitude. Therefore, the implementation of pattern and relational training in early education could help improve future mathematical performance, thus warranting further study. This study involves 4- to 6-year-old children. In Experiment 1, we explored how the use of perceptually rich stimuli (i.e., everyday objects) compared to the use of more impoverished stimuli when teaching children patterning. The study utilized a basic pre- to post-test design, in which children were trained with either perceptually rich materials or more perceptually impoverished materials, followed by measures of generalization and transfer. Children trained on simple shapes outperformed those who were trained on "real objects," such that children were more likely to correctly choose the pattern match in the more impoverished condition. Experiment 2 was developed based on the results of Experiment 1 and utilized simplistic stimuli to train children on pattern matching in one of two formats: massed (all training together) or spaced (breaks during training). Here, we found that children learned the pattern match across both conditions; however, gains following a two-week delay were only evident in the Massed condition. Together, results from this study suggest that young children conceptualize patterning

better when simple materials are used, however it is unclear whether massed or spaced training is better for pattern learning.

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The implications of patterning skills on future mathematical aptitude are important given national achievement data in the United States. The National Assessment of Educational Progress reported that less than half of fourth and eighth graders performed at or above a proficient level in mathematics (Gentner & Medina, 1988; NCES, 2015; Son, Smith, & Goldstone, 2011). Furthermore, national mathematics scores were lower in 2015 than they were in 2013 (NCES, 2015). These results not only point to deficiencies in the American educational system, but also indicate a failure to remedy these issues over time. The implementation of pattern teaching in preschools as part of a standardized curriculum could help improve future mathematical achievement of American children, and thus the future of mathematical achievement across an individual's lifespan.

A human's ability to recognize patterns in early development is predictive of later math achievement (Rittle-Johnson, Fyfe, Hofer, & Farran, 2016; Sarama & Clements, 2009). In other words, the development of patterning logic is an antecedent to understanding mathematics. Patterning, which is defined as identifying a predictable sequence, is often introduced to children in the form of a repeating linear set, such as a string of two alternating colors (e.g., red-blue-red-blue; Rittle-Johnson et. al, 2016). Patterning skills rely on the detection and application of an underlying rule (e.g., category 1- category 2 - category 1- category 2) to new situations, which is an imperative skill needed for success in mathematics (NCTM, 2000; Rittle-Johnson et al., 2016; Steen, 1988). Research has shown that skill with patterning at age seven is a strong predictor of mathematics knowledge at age 11 (Rittle-Johnson et. al, 2016). In addition, early pattern learning is especially relevant for sub-disciplines of math such as algebra, which involve detecting commonalities in a set (similar to pattern learning; NCTM, 2000; Rittle-Johnson et al., 2016; Steen, 1988) and applying these commonalities to new math problems. Furthermore, patterning ability is linked to more proficient spatial skills, executive function skills, fluid reasoning, and reading development (Kidd et al., 2014; Pasnak et al., 2016; Burgoyne et al., 2017; Colins and Laski 2015; Rittle-Johnson, Fyfe, Mclean, & McEldoon, 2013; Rittle-Johnson, Zippert, & Boice, 2018).

The potential benefits of patterning can be inaccessible for many children, as inherent cognitive limitations exist. Most notably, young children struggle with processing and generalizing relational information, which is imperative for understanding patterns. In one study, Gentner and Medina (1988), found that children younger than 6 years of age have difficulty noticing when sets share a common relationship among objects (pattern match) because they focus on the identity of the objects themselves (e.g., the correct match to the pattern “big square-small square” would be the pattern “big circle-small circle” because the relationship between objects in a set is a *big version* and *a small version* of the same shape, however, children will instead pick an identity match, such as two squares of the same size; also see Kotovsky & Gentner, 1996). Results from this line of work suggest that young children may have difficulties noticing complex or novel patterns due to the fact that their attention is object-based (Kanwisher & Driver, 1992), as seen in other mathematical tasks at this stage of development.

Moreover, children commonly lack the selective attention necessary to attend to relevant pattern features, particularly in their preschool years. This has been attributed to inefficient information filtering and distributed attention. In instances where extraneous information is present, preschoolers’ inability to filter out superfluous information may be taxing on working memory, which further reduces processing of pertinent information. Enns and Akhtar (1989) found that 4- and 5-year old participants were less accurate on a flanker task (where they were asked to determine the direction of an arrow) when compared to 7-year olds and adults because they were more susceptible to influence by distractors when completing the task. This and additional cognitive data suggest that young children have difficulty filtering out irrelevant information – despite their familiarity with the items themselves – and that this filtering is poorer in 4-5 year olds, as compared to 7 year olds (also see Deng & Sloutsky, 2015, 2016; Plebanek & Sloutsky, 2017).

Together, children's limited ability to reason relationally combined with their limited ability to filter out irrelevant information when completing cognitive tasks, suggests that more attention should be paid to how we teach young children novel mathematics concepts and, in particular, how we teach them about patterning. Furthermore, these difficulties may preclude children from early algebraic understanding (in which they must reason abstractly, which is often relational in nature), suggesting that an inability to understand relations (such as patterns) early on may also affect how children generalize and transfer that knowledge down the line. Therefore, facilitating pattern knowledge and retention early in development, as well as the mechanism for that success, is critical.

Current Study

Understanding the mechanism behind successful pattern learning early in development can help parents and educators choose the correct materials when teaching young children critical math skills. This is the focus of the present study. Critically, we ask two research questions across two experiments in an effort to develop and assess the manner in which children learn about (and subsequently generalize and transfer) patterns.

Experiment 1: *Are perceptually impoverished or perceptually rich materials better for pattern learning, generalization, and transfer?* This question is particularly relevant given that teachers and parents tend to prefer non-idealized (perceptually rich) materials when engaging children in broader math learning. When Petersen and McNeil (2013) surveyed teachers from local childcare centers, they asked what teaching aid they would prefer for sorting and counting lessons. Teachers preferred brightly colored, realistic plastic animals over flat solid-colored disks. Similarly, even teaching supply stores sell a wide variety of perceptually rich learning aids with the idea that they will be more exciting for students (Petersen & McNeil, 2013). Educators' preference for perceptually rich learning aids may center on the belief that they will encourage engagement and motivate retained attention and subsequent learning (Petersen & McNeil, 2013).

Despite some research suggesting that teachers and educators may prefer vibrant and eye-catching stimuli when teaching children about novel math concepts, other evidence suggests that this may not be best for young learners. In reality, perceptually interesting or vibrant information may be distracting if it is not the focus of the to-be-learned content given young children's stage of attention development. Furthermore, research in other domains of mathematics suggests that "less is more" when teaching children about difficult or novel concepts. This work suggests that perceptually-rich or concrete materials may hinder learning (and perhaps problem-solving and computation) because extraneous perceptual information gets integrated into the representation of the target concept (e.g., Kaminski & Sloutsky, 2009, 2013; Kaminski, Sloutsky, & Heckler, 2009; McNeil & Fyfe, 2012; Mix, 1999, 2008; Peterson & McNeil, 2012; Posid & Cordes, 2015).

To expand upon this, in a study by Kaminski and Sloutsky (2013), young children were taught how to read bar graphs. Children who were taught using bar graphs that had attention-grabbing visuals (e.g., flowers, basketballs, etc.), rather than monochromatic bar graphs, often focused on irrelevant details, incorrectly trying to use them as a tool to read the bar graphs (Kaminski & Sloutsky, 2013). In another example, Posid and Cordes (2014) found that adding more detailed, varied stimuli (heterogeneous arrays) was detrimental to children's ability to count items in which they had to identify which of two arrays contained a target number (Posid and Cordes, 2014). Finally, Kaminski and Sloutsky (2009) taught first-grade children how to label objects within a set as a fraction in numerical form. The researchers found that participants who were taught how to label proportions using black and white circles performed significantly better on learning and transfer than those who were taught using pictures of colored flowers (Kaminski & Sloutsky, 2009). They concluded that the concrete examples hindered initial learning, which, in turn, affected the participants' ability to apply the correct numerical information to future trials (Kaminski & Sloutsky, 2009). This work suggests that when learning novel math concepts, simple materials can aid in acquisition, while more detailed materials can cause interference.

Despite the previously described work suggesting the impact of perceptual properties on math learning and generalization, almost no studies have specifically examined the impact of perceptual presentation on children's ability to learn and generalize novel pattern information. Thus, although some factors facilitating pattern learning are known, the primary goal of Experiment 1 was to pit perceptually simple vs. perceptually rich stimuli against each other as a means to promote pattern learning, generalization, transfer, and long-term retention in preschool-aged children. We examined this factor by training children on novel patterns using either simple shapes or real-world objects (see Figure 1). Although research suggests that perceptually rich and variable educational materials surround a child's learning environment (e.g., Van de Walle, 2007), presumably because these high-contrast items are attention-grabbing and motivating, this may be detrimental to children's learning (McNeil et al., 2009; Petersen & McNeil, 2013). Thus, we hypothesized that children trained on simple shapes, rather than engaging, real-world objects, would more accurately select a pattern match in our task following training.

Experiment 2: *Does massed vs. spaced practice during pattern instruction best facilitate the retention of pattern knowledge?* Following results from Experiment 1, stimuli from the condition which provides the highest learning, generalization, and transfer will be used to develop a two-day version of Experiment 1's design, so as to measure retention of the learned pattern information. Additionally, training will be manipulated such that children will receive either massed practice (all trials presented together, as in Experiment 1) or spaced practice (trials will be interrupted twice to give children a short "break" from the to-be-learned materials). Previous research suggests that spacing out instances of practice results in better learning outcomes than when new material is engaged without breaks (Cepeda, Pashler, Vul, Wixted, & Rohrer, 2006; Rohrer & Taylor, 2006).

In fact, "spacing" is intuitively how most curriculums are designed. That is, children do not learn a subject for eight straight hours, but may space a one-hour lesson over several days, or even weeks. However, this is a relatively unexplored area in mathematical research, with most of the research on spaced vs. massed practice involving broader memory and cognition. Additionally, these prior studies have typically studied adults. The few studies that do involve children have, again, been far outside the realm of math or pattern learning. Therefore, we seek to investigate these two methods of training within the context of children's learning and retention of patterning skills. Which type of training will promote better learning, generalization, and transfer, as well as long-term retention (2 weeks later)? We predict that children trained via spaced practice will show higher accuracy on pattern matching on the day of training. Although no research, to our knowledge, serves as a basis for predicting retention at Day 2 (2 weeks after initial training), we hypothesize that any benefit of spaced practice at Day 1 will hold for Day 2.

EXPERIMENT 1

Methods

Participants

Eighty-four 4- to 6-year-old children participated in this study (M=4.87 years, SD=0.65 years). Children were run at one of two testing locations, either at their local preschools or daycare centers near Columbus, Ohio, or during a single visit to The Ohio State University Cognitive Development Lab. Participants were assigned to one of two training conditions (Simple Shapes: n=34, M=4.97 years, SD=0.58 years; Real Objects: n=50, M=4.80 years, SD=0.70 years). Five participants failed to complete the task and their data was excluded from all data analyses.

Materials

The materials used in the Pattern Task were pictures of shapes arranged in a linear pattern (based on Fyfe et al., 2015). This task was a match-to-sample task, unless otherwise described, in which participants were asked to look at an exemplar (top center) and choose from one of two side-by-side patterns below (see Figure 1). Critically, the side-by-side "choice" patterns were either a pattern match (same pattern as exemplar but different shape or color) or a perceptual match (different pattern as exemplar but same shape or color; see Figures 1 and 2). The Pattern Task consisted of seven blocks of trials: Warm-up, Pretest, Training, Posttest, Generalization-4, Generalization-conflict, and Transfer (see Figure 2). All blocks of the experiment were identical for both conditions, except for Training (described below). For all trials, participants saw the stimuli in the same pre-determined order. To control for children's knowledge of shapes and colors, half of all trials were color matches (shape is not a factor when identifying the correct pattern match) and half of all trials were shape matches (color is not a factor when identifying the correct pattern match; see Figure 3).

Warm-up consisted of four unscored practice trials. Participants were shown a sample pattern in the top center of the screen made up of two colored shapes and were asked to choose the correct match (same color and shape pattern) from two sample choices at the bottom of the screen. The patterns shown were A-A (e.g. red pentagon – red pentagon, two trials) and A-B (blue circle – yellow circle, two trials) for a total of four warm-up trials.

Following the Warm-up, participants completed a pretest, which consisted of 12 trials. Again, participants were shown an exemplar at the top center of their screen (made up of three items) and were asked to select from two samples at the bottom of their screen. The patterns shown were A-B-B, A-A-B, A-B-A, and B-B-A. Each pattern was presented three times for a total of 12 trials.

Training consisted of eight sets of three trials for a total of 24 trials (8 unscored examples and 16 scored trials). Each set of trials consisted of an "example" slide (modeled from Fyfe et al., 2015) which contained two examples of the same pattern (e.g., top pattern: A-A-B configured as a blue square – blue square – red square; bottom pattern: A-A-B configured as blue circle – blue circle – red circle). After an example, two "solve" trials were presented. Each followed the match-to-sample appearance of Pretest, displaying an exemplar in the top center and two sample pattern choices below from which children had to select the pattern match. The same pattern (e.g., A-A-B) was used across a set of example-solve-solve slides, but the pattern varied across sets (A-B-B, A-A-B, A-B-A, and B-B-A, two presentations of each pattern set).

Training was the only block of trials in which the stimuli differed by condition. The Simple Shapes condition consisted of color and shape stimuli, resembling those used in pretest and posttest. The Real Objects condition consisted of pictures of everyday items that children would be familiar with (e.g., animals, plants, furniture, vehicles, toys, food, and other objects one might encounter in his or her environment; see Figure 1). Across the two conditions, children saw identical patterns, such that only the stimuli, not the pattern or procedure, differed.

The training was followed by a posttest that was identical to Pretest (12 trials).

After the posttest, there were two measures of generalization. The first series of trials were called Generalization-4 and stimuli were similar to those used in the pre/post-test (see Figure 4). The difference was that four items made up each pattern, rather than three. Again, participants were shown an exemplar at the top center of their screen and were asked to select from two samples at the bottom of their screen. The patterns shown were A-A-A-B, A-A-B-B, A-B-A-B, and A-B-B-B. Each pattern was presented three times for a total of 16 trials.

Generalization-4 was followed by a different type of generalization block called Generalization-conflict. Generalization-conflict trials were match-to-sample items made up of three items (similar to pre/post-test), but crossed two dimensions (both shape and color, rather than a single dimension) such that children had to ignore both perceptual features (shape and color) to identify the correct pattern match (see Figure 4). The patterns shown were A-B-B, A-A-B, A-B-A, and B-B-A. Each pattern was presented four times for a total of 12 trials.

The last set of trials were a measure of transfer. Instead of a match-to-sample task, the Transfer task was a fill-in-the-blank task. The proper completion of Transfer was meant to signify a deeper understanding of patterns and the ability to apply that knowledge to a novel instance. Here, participants were shown an exemplar at the top center of their screen and a second exemplar directly underneath, in which one item was replaced with a question mark symbol (i.e., top pattern: pink star – pink triangle – pink star – pink triangle; bottom pattern: orange heart– green heart – ? – green heart). Four answer choices (i.e., green heart, orange heart, pink heart, black heart) were presented at the bottom of the screen and participants were asked to fill in the blank to make their pattern the same as the top pattern (see Figure 4). The patterns shown were: A-B-A-B, A-A-B-B, A-A-A-B, and A-B-B-B. Each pattern was presented twice for a total of eight transfer trials.

Per research suggesting the impact of relational language on pattern learning (that is, training on “A-B-A” is more beneficial than training on “red-blue-red”; Fyfe et al., 2015; Posid et al., 2018; Rittle-Johnson et al., 2016), half of participants heard and saw labels that were abstract (“A-B-A”) and half of participants heard and saw labels that were concrete (“red-blue-red”). However, initial analyses indicated no interactions (*ps*>.4) or main effects (*ps*>.1) of this variable, so it was not included in subsequent data analyses.

Procedure

Children took part in this study at one of two locations, either during a single testing session at their school or during a single visit to the laboratory on campus. All children were tested in a quiet area by a female experimenter. Children completed the experiment on a 13-inch Macbook Pro and answers were recorded by the experimenter. The same procedure was used across both the Simple Shapes and Real Objects conditions.

The study consisted of 92 trials in total that children completed in a single session. The first block of the experiment was a warm-up. During Warm-up, the researcher gave an explanation of the two-item pattern and the child was asked to find the same pattern from two choices. The research assistant said, for example, “The part that repeats in my pattern is A-A because it has two that are the same. Can you find the same pattern?" Feedback regarding whether the participant made the correct choice or not was given during the Warm-up block only and answers were not scored nor included in any subsequent data analyses.

In Pretest, the researcher said (and this text was displayed on the computer screen as well), "Can you find the same pattern?" without any further explanation. Children selected their answer by pointing at the screen and the experimenter recorded their answer. Children were not given corrective feedback.

In the Training block, the researcher used the example trial (first of three trials in a set) to explain why the two sample sequences were the same pattern; for example, "The part that repeats in the top pattern is A-B-A because it has one, then one that is different, then one that is the same as the first. The part that repeats in the bottom pattern is also A-B-A because it also has one, then one that is different, then one that is the same as the first. These patterns are the same because the secret code for both patterns is A-B-A." Children did not need to make a selection on this example trial and the experimenter manually moved to the next trial when they had finished speaking. Following this example, there were two match-to-sample trials where participants were asked to find the pattern match when given a sample pattern and two answer choices. These instructions were, "The part that repeats in my pattern is A-B-A. Can you find the same pattern?" for example. Children selected their answer and were not given corrective feedback.

After Training, participants were taken through two generalization blocks: Generalization-4 and Generalization-conflict. The research assistant gave the same instructions as those for Pretest and Posttest ("Can you find the same pattern?") and children selected their answer with no corrective feedback.

During the Transfer block, the researcher explained the top pattern by saying, for example, "The part that repeats in my pattern is A-B-A-B. Can you fill in the blank for your pattern?" The researcher then pointed to the blank space (represented with a question mark symbol) that needed to be "filled in" and motioned to the four possible answers while saying the previous statement. Again, children's answers were recorded and they did not receive corrective feedback. Children were rewarded with stickers after each block of the experiment regardless of performance on the task in order to encourage neutral but positive continued participation.

Results and Discussion

We ran an initial univariate ANOVA to examine the impact of condition (2: Simple Shapes vs. Real Objects) on overall accuracy in the patterns task. Results revealed a significant main effect of condition ($F(1,84) = 13.7$, $p < 0.001$, $\eta_p^2 = 0.82$) indicating that children were more likely to correctly choose the pattern match in the Simple Shapes condition ($M = 62.0\%$) compared to the Real Objects condition ($M = 41.3\%$; see Figure 5). Critically, children in the Simple Shapes condition were more likely to select the pattern match (vs. 50%: $t(35) = 3.17$, $p = 0.003$, *Cohen's d* = 1.07) and children in the Real Objects condition were more likely to select the perceptual match (vs. 50%: $t(49) = -2.25$, $p = 0.029$, *Cohen's d* = 0.64).

We were also interested in whether Condition interacted with Phase of the study. Results from a 2 (Condition) X 6 (Phase) Repeated Measures ANOVA revealed a significant interaction between Phase and Condition (F(5, 420)=5.55, p<.001, η_p^2=.062; Figure 5).

Follow-up independent samples t-tests indicated that there was no significant difference ($t(84) = 1.01$, $p = 0.317$, Cohen's $d = 0.22$) between the Simple Shapes condition ($M = 45.2\%$) and the Real Objects condition ($M = 38.7\%$), indicating that children performed equally across conditions prior to our training intervention.

There was a significant difference between conditions during Training ($t(84) = 3.49$, $p = 0.001$, Cohen's $d = 0.78$), with children performing better in the Simple Shapes condition ($M = 75.7\%$) than in the Real Objects condition ($M = 48.8\%$). During Training, children were more likely to choose the pattern match over the perceptual match ($t(35) = 5.18$, $p < 0.001$, Cohen's $d = 1.75$) in the Simple Shapes condition, but chose at chance-level in the Real Objects condition ($t(49) = 0.227$, $p = 0.018$, Cohen's $d = 0.11$).

Like Training, Posttest also showed a significant difference between condition ($t(84) = 4.77$, $p = 0.001$, Cohen's $d = 1.06$). During Posttest, when participants were no longer given explicit instructions, children in the Simple Shapes condition ($M =74.5\%$) continued to choose the pattern match (vs. chance: $t(35) = 5.03$, $p < 0.001$, Cohen's $d = 1.70$), whereas children in the Real Objects condition ($M =39.2\%$) reverted back to choosing the perceptual match (vs. chance: $t(49) = 2.07$, $p = 0.043$, Cohen's $d = 0.59$).

Accuracy across both generalization blocks of the experiment showed a significant difference between conditions (Generalization-4: $t(84) = 3.99$, $p < 0.001$, Cohen's $d = 0.90$; Generalization-conflict: $t(84) = 2.18$, $p =0.032$, Cohen's $d = 0.47$). On Generalization-4 trials, children were better at selecting the pattern match in the Simple Shapes condition ($M = 69.3\%$) and did so at an above-chance level ($t(35) = 4.39$, $p < 0.001$, Cohen's $d = 1.48$), whereas children in the Real Objects condition continued to select the perceptual match ($M = 40.1\%$; vs. chance: $t(49) = 1.85$, $p = 0.07$, Cohen's $d = 0.53$). Condition differences and an advantage for Simple Shapes also emerged in Generalization-conflict trials, although there was broadly lower performance in these much more difficult trials (Simple Shapes condition: $M = 41.3\%$, vs. chance: $p>.1$); Real Objects condition: $M = 24.3\%$, vs. chance: $t(49) = 5.46$, $p < 0.001$, Cohen's $d = 2.5$).

Results showed no significant effect of training condition on the novel Transfer block of trials ($t(84) = 1.53$, $p = 0.129$, Cohen's $d = 0.33$), indicating that participants performed equally across conditions (Simple Shapes condition: $M = 51.7\%$; Real Objects condition: $M = 42.5\%$), albeit above chance-level (Simple Shapes: vs. 25%: $t(35) = 0.37$, $p = 0.713$, Cohen's $d = 0.13$; Real Objects: vs. 25%: ($t(49) = 1.95$, $p = 0.057$, Cohen's $d = 0.89$).

Secondary analyses specifically examined participants' gains (that is, any improvement between pretest and posttest or generalization; measured as a difference score: Posttest minus Pretest or Generalization-4 minus Pretest) and whether they differed as a function of condition or age. A Repeated Measures ANOVA indicated a significant interaction between Gains and Condition ($F(1, 78) = 4.36$, $p = 0.04$, $\eta_p^2 = 0.053$), with greater gains from pretest to both posttest (M=29.3%, $t(35) = 6.31$, $p < 0.001$, *Cohen's d* = 2.13) and generalization (M=24.0%, $t(35) = 5.53$, $p < 0.001$, *Cohen's d* = 1.87) in the Simple Shapes condition, but not in the Real Objects condition (*Ms*<.01, *ps*>.7; see Figure 6).

In sum, although preschool-aged children initially do not discriminate between a pattern vs. perceptual match (pretest), explicit training in the Simple Shapes condition promoted their ability to make a pattern match, rather than a perceptual match, which carried through both posttest and generalization. In contrast, children in the Real Objects condition were more likely to pick the perceptual match rather than the pattern match following training. This suggests that the use of perceptually rich and "exciting" items, such as toys and animals, are distracting for young children and bolster their focus on the items themselves, rather than the relation between them.

Additionally, several limitations emerge from this experiment. First, this experiment took place over a single testing session, so it is unknown whether any benefits of training would carry over past a single day; that is, was training sufficient for children to maintain its benefits or do these effects taper without subsequent training sessions? Second, although children in the Simple Shapes condition were more likely to pick the pattern match during and following Training, this did not hold for the most difficult trials (Generalization-Conflict) and, broadly, children were not performing near ceiling-level. This suggests that, although Simple Shapes may help bolster children's focus on relations between sets as in our match-to-sample patterns task, additional information could bolster this accuracy even more. Experiment 2 explores both of these limitations and remaining open questions.

EXPERIMENT 2

The results of Experiment 1 demonstrated an advantage of using simple shapes (rather than perceptually rich items) to teach children about patterns, so these simple shapes were employed throughout Experiment 2. While Experiment 1 investigated whether the type of stimuli used during training impacted children's ability to learn and generalize new pattern knowledge, Experiment 2 explored whether the type of training itself impacted this learning and subsequent retention. Specifically, we asked whether massed vs. spaced practice at training would impact children's learning, generalization, and transfer of novel pattern knowledge. Additionally, we asked whether children retain any of the new pattern knowledge they gained during their training session by adding a subsequent testing day, two weeks following participants' initial session.

Limited previous work has compared the effects of massed practice versus spaced practice on information recall. Cepeda et al. (2006) conducted a meta-analysis comparing massed presentation (defined in the analysis as a continuous presentation of an item or a lag of less than 1 second) to spaced presentation (defined in the analysis as a case in which study episodes are separated by a lag of 1 second or longer). The analysis showed that participants in the spaced conditions were more accurate at follow-up than participants in massed conditions for all retention interval durations (the time between the final study episode and the final recall test; Cepeda et al., 2006). Spaced practice has also been shown to benefit attention. Ariga and Lleras (2011) examined the impact of “a mental break” on the accuracy of college-age participants during a vigilance task (Ariga & Lleras, 2011). The researchers found that participants who were asked to identify an additional element that was sporadically presented throughout the task (the “switch condition”) performed better than participants whose vigilance task was not interrupted (Ariga & Lleras, 2011). These results suggest that because the participants were able to take a cognitive break during the “switch condition,” they were better able to focus when they resumed the vigilance task (thus, improving accuracy; Ariga & Lleras, 2011). These findings suggest that spaced practice might also have an advantage over massed practice in the context of initial learning, again suggesting an advantage of spaced training over massed training.

In one of the single examples of massed vs. spaced practice on *children’s* learning, Vlach, Sandhofer, and Bjork (2014) compared the impact of either equally spaced learning intervals or gradually increased learning intervals on preschoolers’ retention during a categorization and generalization task. Although there was no difference between these conditions for children tested immediately after the presentation, expanding time schedule was better for long-term generalization (one day later; Vlach et al., 2014). The researchers attributed this enhanced retention to the way in which longer and longer delays in information presentation could induce more forgetting, but that this would-be decrement subsequently caused learners to be more engaged in active retrieval (Vlach et al., 2014).

As it relates more specifically to mathematic knowledge, Rohrer and Taylor (2006) found that college students who practiced math problems in a distributed fashion (spaced), in which the problems were separated into two sessions of 5 math problems (one week apart), performed 32% more accurately on a math test four weeks later. In comparison, their peers who practiced all 10 problems in one sitting (massed), were less accurate after the four weeks.

Based on these initial findings, we hypothesize that children will be more likely to pick a pattern match following spaced practice, as compared to massed practice, during pattern training and that this advantage will be retained within the same session and following a two-week delay. In the context of patterning, when the experimental sessions are separated by a break or an alternate task, children should be less fatigued when they are engaging in the second session.

Methods

Participants

Thirty different children participated in Experiment 2 (M=4.80 years, SD=0.61 years). These children were tested at local schools or at The Ohio State University Cognitive Development Lab. Participants were randomly assigned to one of two conditions: Massed Practice (n=17, M=4.82 years, SD=0.53 years) or Spaced Practice (n=13, M=4.77 years, SD=0.73 years). Due to the two-week delay, 13 participants were lost to follow-up (i.e., they participated in Day 1, but not Day 2, of the study).

Materials

The materials used in Experiment 2 were identical to those used in the Simple Shapes condition of Experiment 1, with the following noted differences. As in Experiment 1, Experiment 2 consisted of the same seven blocks: Warm-up, Pretest, Training, Posttest, Generalization-4, Generalization-conflict, and Transfer. However, in Experiment 2, the number of Training trials was doubled, such that each of the four patterns was presented four times. This made Training more extensive with a total of 16 examples and 32 trials in which the child had to select the correct pattern match.

Pattern Task. Day 1 of the Pattern Task was presented as follows: In the Massed condition, children completed all Training trials in a row, as in Experiment 1. In the Spaced condition, children completed half of the Training trials, followed by a short break, then completed the second half of the Training trials, followed by a short break.

The two breaks were filled with pre-determined numeracy tasks (see Posid & Cordes, 2014, 2015; Posid, Huguenel, & Cordes, 2013): an Estimation Game and counting Card Task). In the Massed condition, these games were completed at the end of the Pattern Task, following completion of the Transfer trials.

Day 2 of the Pattern Task was completed approximately two weeks following the initial session and presented children with the exact same Posttest, Generalization-4, Generalization-conflict, and Transfer trials as appeared in Experiment 1 and Day 1 of Experiment 2. Children did not receive any warm-up or training on Day 2, and did not receive any corrective feedback throughout the task. Day 2 did not differ based on Massed vs. Spaced condition placement.

Numeracy Tasks. The Estimation Game (based on Posid & Cordes, 2014; Posid et al., 2013) was played on the same Macbook laptop and consisted of two tasks, completed one right after the other. The first of these tasks was a numerical discrimination game, presented children with two side-by-side arrays that were either both homogenous (all red circles) or both heterogeneous (varied in color and shape; see Figure 7A). The items were randomly placed to deter children from trying to count the items. Element size was kept constant within each array, but varied between arrays. There were 24 test trials consisting of small (4v6, 4v8), medium (8v12, 8v16), and large (16v24, 16v32) arrays. Children did not receive corrective feedback.

The second task in the Estimation Game presented children with a single array of shapes and children were prompted to verbally estimate the number of items in the array (see Figure 7B). The arrays were similar to those in the discrimination task, in that half of them were homogeneous in make-up and half of them were heterogeneous in make-up. There were 2 practice trials and 10 scored test trials. Children did not receive corrective feedback.

The Card Task (Posid & Cordes, 2015) was displayed on the same Macbook laptop. Here, children saw two side by side arrays of animals that were homogenous in make-up (e.g., 9 flamingos vs. 6 flamingos; see Figure 8). In this task, children were prompted to select which of the two sides contained six items. Unlike in the Estimation Game, the items in the Card Task were laid out in countable configurations to encourage children to count the items individually. Presentation time was indefinite and participants did not move to the next trial until they had selected their answer. The Card Task contained 8 scored trials and children did not receive corrective feedback.

None of the numeracy tasks were played as part of the Day 2 follow-up.

Procedure

Pattern Task. As in Experiment 1, children took part at one of two locations, either during testing sessions at their school or during a visit to the laboratory on campus. All children were tested in a quiet area by a female experimenter. Children completed the experiment on a 13-inch Macbook Pro and answers were recorded by the experimenter. The same procedure as Experiment 1 was used, unless otherwise noted as follows.

Day 1 was completed in a single session. Children completed an identical warm-up and pretest as in Experiment 1. Training for Day 1 of Experiment 2 was identical, except for that the number of trials were doubled. Additionally, participants in the Spaced condition completed the

Estimation Game (described below) after half of the Training trials had been completed and the Card Task (described below) after the other half of the Training trials had been completed. After the Training trials, children in both conditions completed Posttest, Generalization-4, Generalization-conflict, and Transfer trials, which were again identical to Experiment 1. To control for the duration of the study, participants in the Massed condition completed the numeracy tasks as well, but they were completed after Transfer.

On Day 2 (approximately two weeks later), children in both conditions were tested on Posttest, Generalization-4, Generalization-conflict, and Transfer, which were identical to those used in Experiment 1 and Day 1 of Experiment 2.

Numeracy Tasks. Children first played the Estimation Game – either after half of the Training trials had been completed (Spaced condition) or following the completion of the Pattern Task (Massed condition). Stimulus presentation and data recording were controlled by a RealBasic program on a 13-inch MacBook laptop. Children were not given corrective feedback. Additionally, children were specifically instructed not to count, and were reminded not to if they attempted to do so.

During the discrimination task, children were presented with two arrays that were either both homogenous or both heterogeneous in make-up and were asked to indicate which side had the greater number of items. Stimuli were presented for three seconds, followed by the prompt, "Which side had more?" Participants completed four practice trials, followed by 24 scored test trials. Children did not move to the next trial until selecting an answer. Children either verbally indicated their choice or pointed to a side, and the experimenter recorded their responses on the computer.

During the verbal estimation task, children were presented with a single array that was either homogenous or heterogeneous in make-up. The trial was presented for three seconds, followed by the prompt, "How many?" Children were asked to provide a verbal estimate of the number of items presented on the screen. The experimenter recorded the child's response on the computer. Children were reminded not to count but to provide their best guess. Children completed two practice trials, followed by 10 scored test trials.

Children next participated in the Card Task. Here, children saw two side-by-side homogenous arrays made up of familiar animals and were asked to select which side contained six items. Children were always asked to select the same target number. Children provided a verbal response or pointed to indicate their choice and the experimenter recorded their answer.

Results and Discussion

An initial cross-experiment analysis was run to confirm that extending the number of training trials between Experiments 1 and 2 replicated the pattern of results observed in Experiment 1. A Phase (6) X Condition (Simple Shapes, Massed, Spaced) was run and confirmed no main effect of Condition ($p > 0.4$, $\eta_p^2 = 0.023$) nor an interaction between Phase and Condition ($p > 0.1$, $\eta_p^2 = 0.067$; see Figure 9)

Massed vs. Spaced Training:

Analyses were initially run separately for Day 1 and Day 2 of this study. A Phase (6) X Condition (2) Repeated Measures ANOVA on Day 1 data revealed a main effect of Phase ($F(5, 140) = 4.9$, $p < 0.001$, $\eta_p^2 = 0.149$), with accuracy increasing following explicit training. There was no main effect of or interaction with Condition (*ps* > 0.5; see Figure 10). A Phase (4) X Condition (2) Repeated Measures ANOVA on Day 2 data indicated no main effects or interactions (*ps* > 0.2; see Figure 11).

Perhaps of more interest, secondary analyses were run comparing Day 1 and Day 2 performance. A Phase (4) X Day (2) X Condition (2) Repeated Measures ANOVA was run and results revealed a significant interaction between Day and Condition (Massed: M_{Day1} = 47.0% vs. M_{Day2} = 60.6%; Spaced: M_{Day1} = 56.3% vs. M_{Day2} = 46.2%; $F(1, 45) = 5.97$, $p = 0.027$, η_p^2 = 0.285; Figures 12 and 13). This interaction indicated that participants generally increased in accuracy from Day 1 to Day 2 in the Massed condition, whereas the opposite trend emerged following Spaced training. In this vein, children were generally more accurate on Spaced during Day 1 trials following Training (Posttest onward), whereas children were more accurate in the Massed condition on Day 2 compared to the Spaced condition. There were no other main effects or interactions (*ps* > 0.1).

Thus, results from Experiment 2 replicate and extend findings from Experiment 1. That is, Day 1 of Experiment 2 replicates our original finding that pattern training increases preschool-aged children's ability to select pattern matches (over perceptual matches) and that this training promotes continued selection of pattern matches even after explicit training is no longer provided (i.e., posttest onward). Interestingly, children continued to pick the pattern match following a two-week delay, with more gains at two weeks observed in the Massed condition (although gains vs. pretest on Day 1 were observable in both conditions). Thus, although both types of training led children to continue to make pattern matches following a delay – suggesting retention of this novel pattern knowledge following training – there seemed to be a slight advantage following the Spaced training on Day 1, but a significant advantage for those in the Massed training following a delay.

Patterning and Math Ability:

Due to the nature of the training task on Day 1, we had a unique opportunity to look at the relationship between patterning and early mathematics understanding. That is, although research to date has found a clear causal relationship between early patterning knowledge and concurrent and later math abilities (Nguyen et al., 2016; Rittle-Johnson, Fyfe, Hofer, & Farran, 2015; Rittle-Johnson, Zippert, & Bolce, 2018), the reverse relationship has not been previously explored, at least to our knowledge.

First, we sought to replicate previous findings that patterning knowledge predicts mathematic ability in early learners. A linear regression examined the impact of pretest pattern knowledge (that is, pattern knowledge prior to training) on Day 1, post-training pattern knowledge on Day 1, post-training pattern knowledge on Day 2, and Condition on accuracy in the Estimation Task. Results revealed that pattern knowledge post-training (*Beta* = 0.999, p = 0.019) significantly predicted estimation accuracy (all other *ps* > 0.06; Model: $R^2 = 0.528$, p = 0.115). A second linear regression examined the impact of the same variables on accuracy in the Card Task. Interestingly, although this model approached significance ($R^2 = 0.524$, $p = 0.065$), no factors individually or significantly predicted success (*ps* > 0.1). Thus, these analyses replicate previous research indicating that early patterning predicts math knowledge (Nguyen et al., 2016; Rittle-Johnson et al., 2015, 2018).

Second, we sought to extend previous literature on the relationship between patterning and math ability by examining the impact of math ability on patterning. A linear regression examined the impact of accuracy on the Card Task, accuracy on the Estimation Task, and Condition on overall patterning accuracy (Day 1). This model was highly significant ($R^2 = 0.805$, $p < 0.001$) and was significantly predicted by accuracy on the Estimation Task (*Beta* = 0.785, p <

0.001; other *ps* > 0.1). These results provide the first evidence that this relationship may be bidirectional; that is, early math knowledge may also be predictive of early patterning knowledge, particularly when numerical abstraction skills are warranted.

General Discussion

The present study investigated the impact of training stimuli and training schedule on pattern learning in preschool-aged children. The aims of this study were to (1) examine whether perceptually rich or perceptually impoverished stimuli are more efficacious for pattern learning, and (2) determine whether massed versus spaced training promotes better retention of learned patterning skills. In Experiment 1, we found that children in the Simple Shapes condition performed significantly better on the patterning task compared to children in the Real Objects condition following training. These results are in line with our prediction that the extraneous perceptual information in the Real Objects condition would be distracting to young children, despite the fact that those items might seem more exciting or engaging. In Experiment 2, we found that participants in the Spaced condition performed slightly better on the patterning task on Day 1, although both types of training promoted pattern matches (rather than perceptual matches), but that participants in the Massed condition were more accurate in selecting the pattern match following a two-week delay.

Experiment 1 demonstrated an advantage of the Simple Shapes condition over the Real Objects condition following training. Patterning relies on the ability to discern relations among objects, rather than focusing on the individual objects themselves. This study suggests that perceptually rich stimuli (real objects) are not only less effective than perceptually impoverished stimuli for pattern learning, but actually make children more likely to choose the incorrect answer when they are prompted to select a pattern match. Therefore, it can be presumed that the extra details highlighted the identities of the individual objects in the patterns and obscured the underlying relations among the objects.

These results align with previous findings that suggest that "less is more" when it comes to teaching children novel mathematic concepts (e.g., Kaminski & Sloutsky, 2009, 2013; Kaminski et al., 2009; McNeil & Fyfe, 2012; Mix, 1999, 2008; Peterson & McNeil, 2012; Posid & Cordes, 2015). In these cases, non-task relevant perceptual information hindered learning because the perceptual features were incorporated into the representation of the target.

In fact, work with preschool-aged children has revealed particular perceptual biases in match-to-sample tasks (Cantlon, Fink, Safford, & Brannon, 2007; Mix, 1999, 2008; Siegel, 1973, 1974; see Posid & Cordes, 2015, for a review), suggesting that early numerical comparisons require high levels of cognitive support since young children's abilities to detect similarity are at least partially derived from perceptual likeness (Mix, 1999, 2008). Given that even infants find perceptual properties of small sets to be at least as salient as number when task demands are ambiguous (Clearfield & Mix, 1999, 2001; Cordes & Brannon, 2009), it is not surprising that young children may be similarly biased to rely upon perceptual attributes under ambiguous circumstances. In fact, even when children are explicitly told that the task is to match sets based on number, children may still be inclined to match sets on multiple dimensions (numerical and non-numerical alike). Therefore, when sets are more simplistic in nature, correct matches to the target relation (in this case, patterns) is much more attainable. For example, one report suggests that children engaged in a numerical same/different task (which requires children to evaluate numerical matches) are more likely to rely upon physical similarities, as opposed to numerical similarities (Defever, Sasanguie, Vanderwaetere, & Reynvoet, 2012). In sum, given that young children's attention tends to be object-based (Kanwisher & Driver, 1992), distributed broadly across both relevant and non-relevant elements (Plebanek & Sloutsky, 2017), and unable to properly filter out unessential features (Enns & Akhtar, 1989), it is not surprising that too much detail might hinder learning, as seen previously in other types of cognitively demanding tasks. The results of our present study extend these findings to the realm of pattern learning and relational knowledge.

In Experiment 2, we sought to extend our findings from Experiment 1 and investigate whether the *delivery* of training affected pattern learning and generalization. We additionally investigated whether pattern training would lead to retention of pattern knowledge two weeks following the initial session. The results from Day 1 indicated that children in the Spaced training condition numerically out-performed children in the Massed training condition, although both conditions trained children to select the pattern match at an above-chance level. In contrast, following a two-week delay, children who had been trained via Massed practice showed higher accuracy and greater gains than those that had been trained via Spaced practice.

This pattern of results is perhaps surprising, given we hypothesized that children would show an advantage following Spaced training compared to Massed training, as some previous research has shown an advantage for learning and retention following spaced practice (e.g., Ariga & Lleras, 2011; Cepeda et al., 2006; Rohrer & Taylor, 2006; Vlach et al., 2014).

Previous research has shown an advantage for spaced practice over massed practice in several situations, but the *degree* of this advantage depends on the combination of inter-study interval and retention interval (Cepeda et al., 2006). On the one hand, Rohrer and Taylor (2006) do not find group differences in accuracy following one week post-practice testing, but do find a difference in accuracy after a 4-week delay. Cepeda et al. (2006) conducted a large meta-analysis and found that when the retention interval was 8-30 days, participants in the spaced condition performed on average 29.4% better than the massed condition group on the final test (Cepeda et al., 2006). This suggests that the length of the retention ratio may be key, with our delay occurring somewhere in the middle of reported findings ranging from no difference to an advantage for Spaced training.

Another possible reason that our findings did not demonstrate an advantage for participants in the Spaced condition is that, according to Cepeda et al.'s meta-analysis, the ideal inter-study interval (or break length) for a retention ratio of about 2 weeks is approximately 1 day (Cepeda et al., 2006). We used a much shorter inter-study interval (5-10 minutes to play a short game) for a retention ratio of 2 weeks. Therefore, these short breaks during training may not have been long enough to promote high retention or to distinguish the training received across the Spaced and Massed condition. Future research should examine both the duration of the delay interval from test to follow-up in combination with the length of the inter-study interval to gauge whether these affect pattern retention, and to what degree they might interact with initial training. Additionally, future research should examine the use of multiple training paradigms (e.g., as seen in Vlach et al., 2014), since only a single training day was included before retention was measured in the present study.

Another possible explanation is that training broken up by the numeracy games (rather than a "true" break, such as sitting quietly or napping) was helpful for short term retention because it helped break up the redundancy of pattern training leading to more engagement and better accuracy for children in the Spaced condition on Day 1. However, it is possible that when children in the Spaced condition had to recall the pattern training two weeks later, memory of the numeracy tasks during training made it more difficult to parcel out the relevant pattern training and learned rules. Future studies should explore if the content of inter-study intervals can interfere with retention.

It should also be noted that Experiment 2 was limited by a small sample size (Day 2: (Massed: n=10, Spaced: n=8). The Pattern Task took approximately 10-20 minutes from start to finish and the numeracy tasks took an additional 10-15. This long exercise made participant recruitment challenging to begin with, and children were ultimately lost to follow-up (a byproduct of testing in schools, where children are absent, sick, etc.).

In conclusion, the present study investigated the most effective stimuli for pattern learning and generalization, and explored whether the presentation schedule (delivery) of novel pattern knowledge impacted children's learning, generalization, or retention of that pattern knowledge. We find that "less is more" when teaching children novel math concepts, in line with data from many previous studies, and extend this to pattern learning and knowledge as well. Thus, we suggest that parents and teachers should not necessarily utilize exciting or engaging learning aids just for the purposes of motivating students, particularly when the skill to be learned is difficult for that age group. We additionally find that children can retain pattern knowledge following a short training session and subsequent delay, although more work is needed to identify the mechanism behind this success. The present study highlights the importance of patterning as an early mathematic precursor and training in this domain may serve to help ameliorate poor mathematic performance in the United States.

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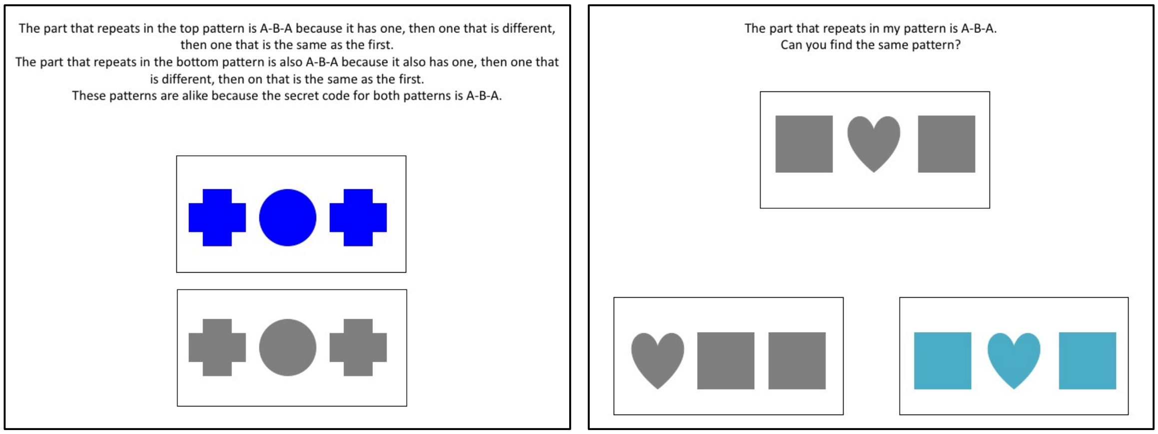
Simple Shapes Condition: “Example” slide and match-to-sample “Solve” slide



Real Objects Condition: “Example” slide and match-to-sample “Solve” slide

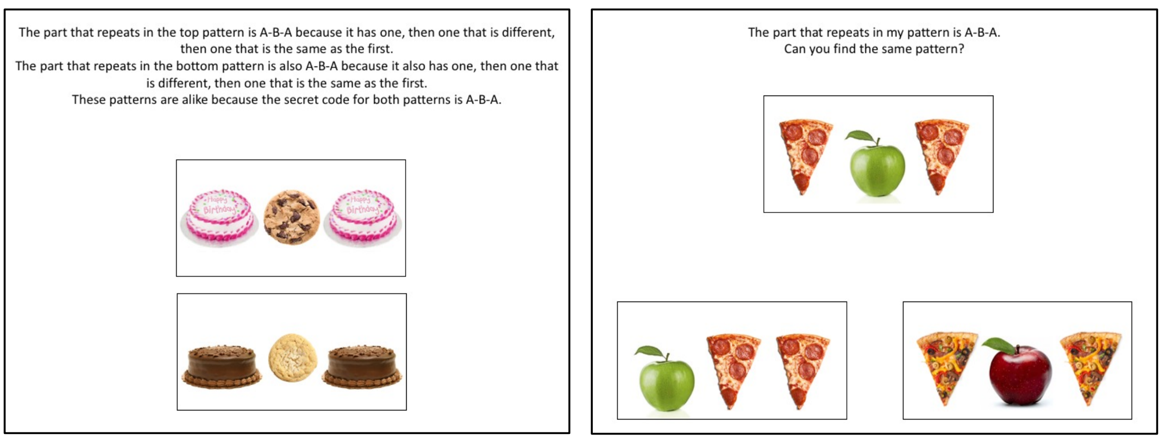


Figure 1. Examples of training slides for both conditions in Experiment 1.

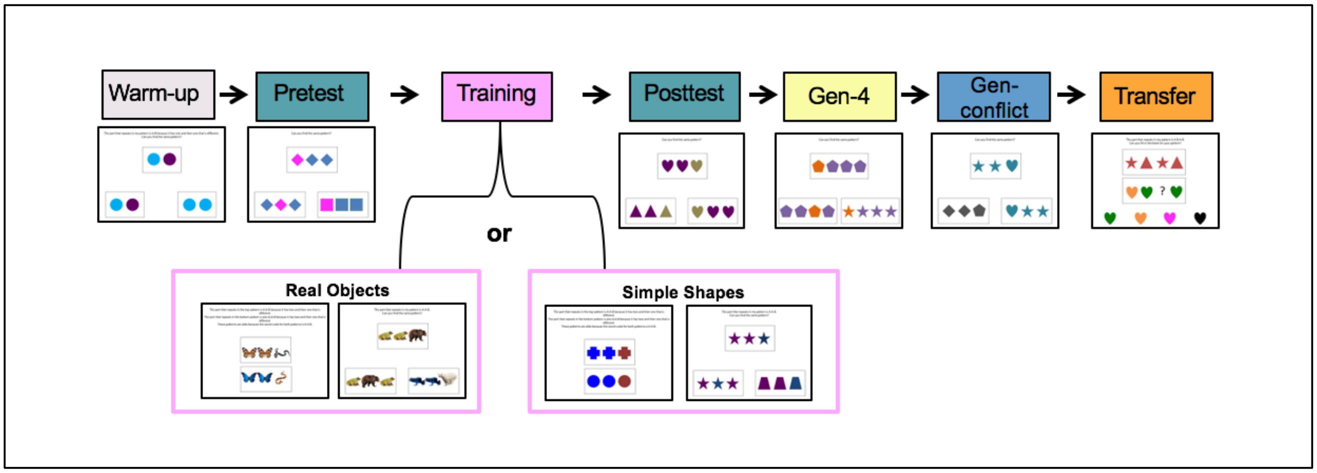


Figure 2. Trial organization for Experiment 1.

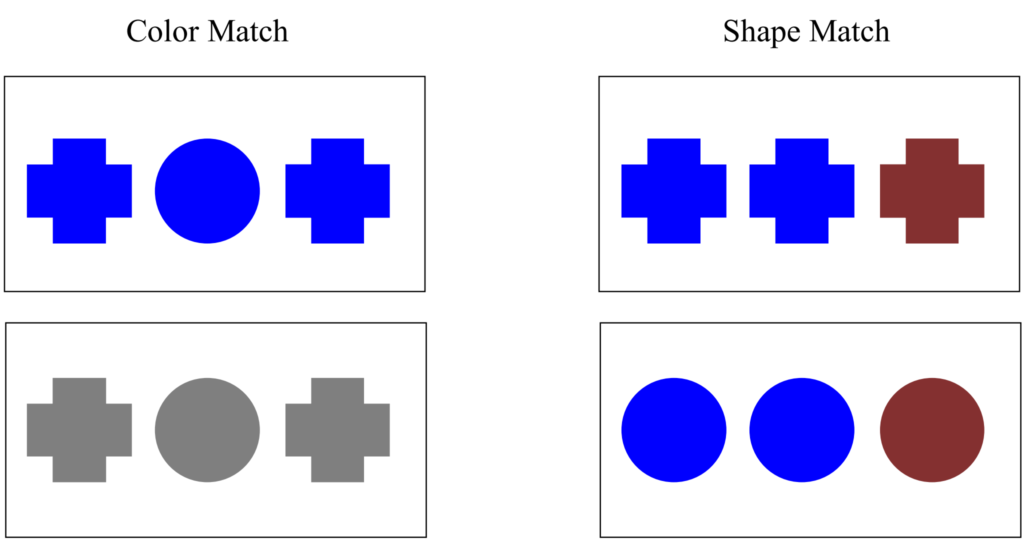


Figure 3. Examples of a color-match trial and a shape-match trial that participants could have seen in Experiment 1. These were randomly intermixed throughout the Patten Task in the Simple Shapes Condition.

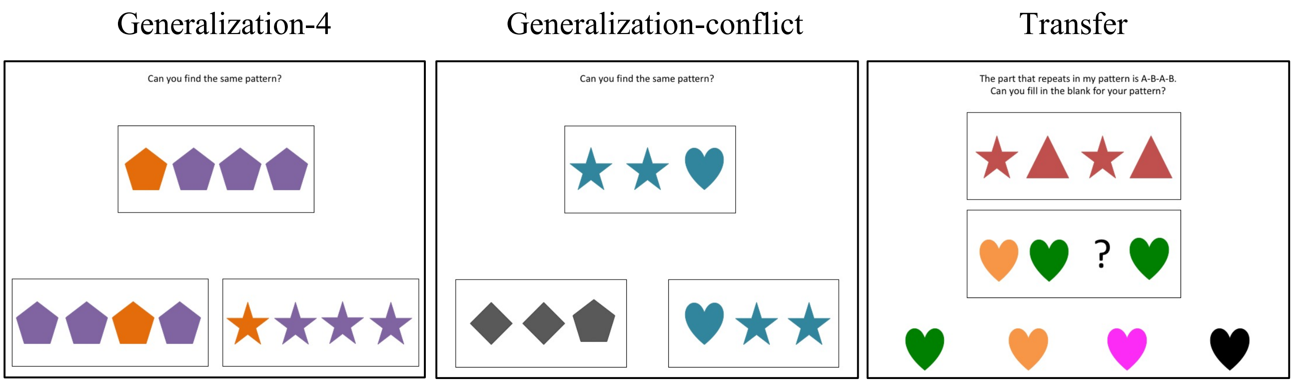


Figure 4. Examples of the stimuli used in the Generalization and Transfer trials of the Pattern Task.

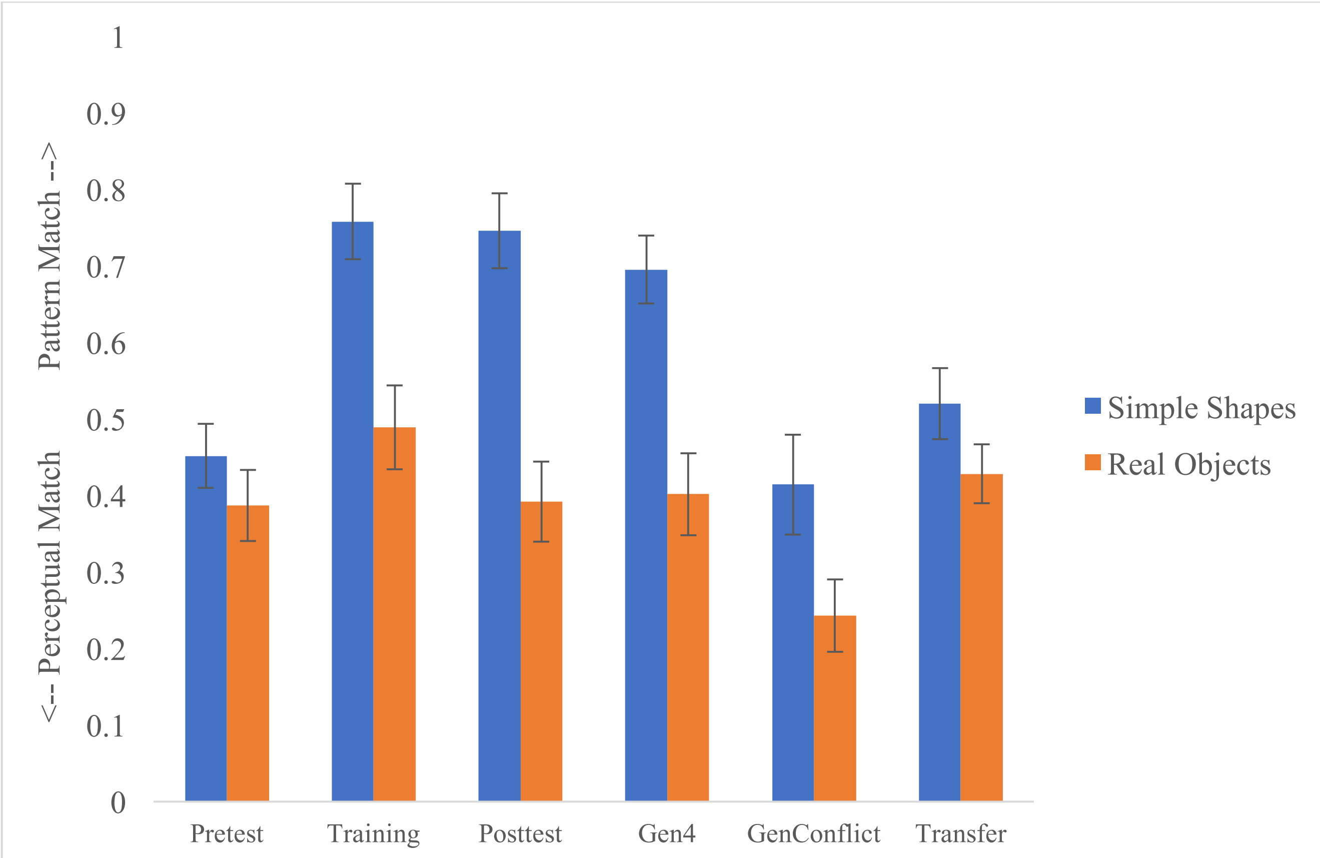


Figure 5. Overall, in Experiment 1, children were more accurate in the Simple Shapes condition compared to the Real Objects condition. This held across each phase of the experiment.

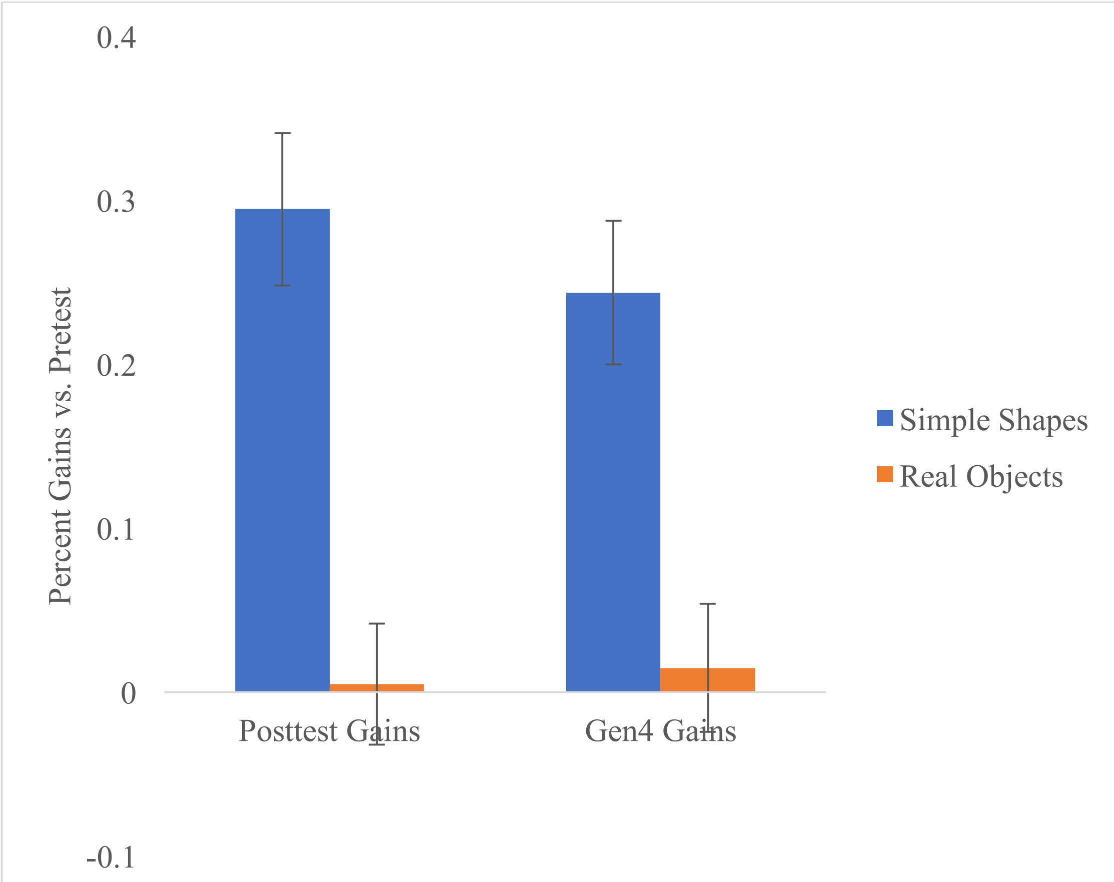


Figure 6. In Experiment 1, children in the Simple Shapes condition demonstrated greater gains in learning from Pretest to Posttest and from Pretest to Generalization-4 as compared to those participants in the Real Objects condition.

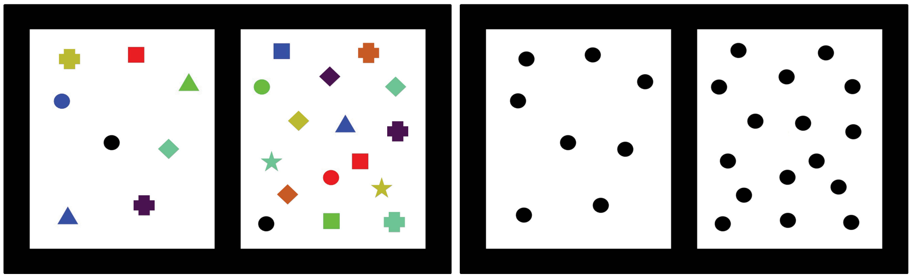


Figure 7A. Examples of stimuli used in the numerical discrimination portion of the Estimation Game (Experiment 2).

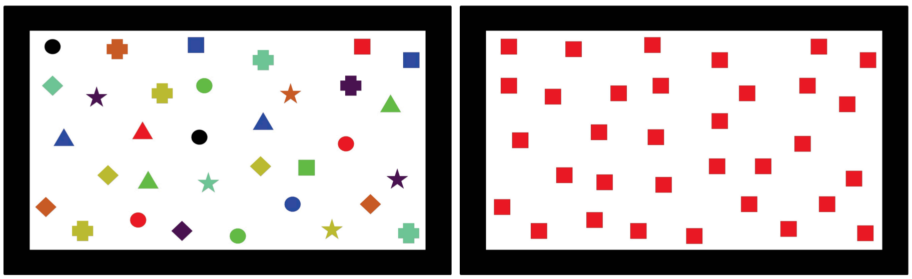


Figure 7B. Examples of stimuli used in the verbal guessing portion of the Estimation Game (Experiment 2).

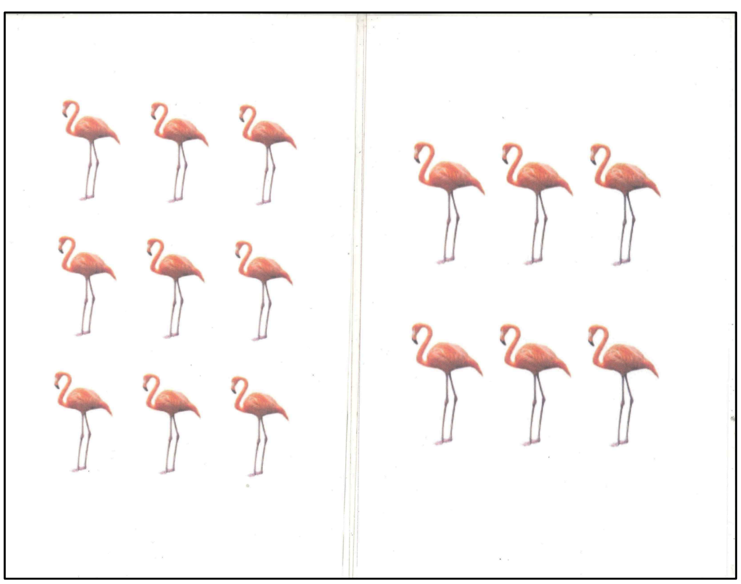


Figure 8. Example of stimuli used in the Card Task (Experiment 2).

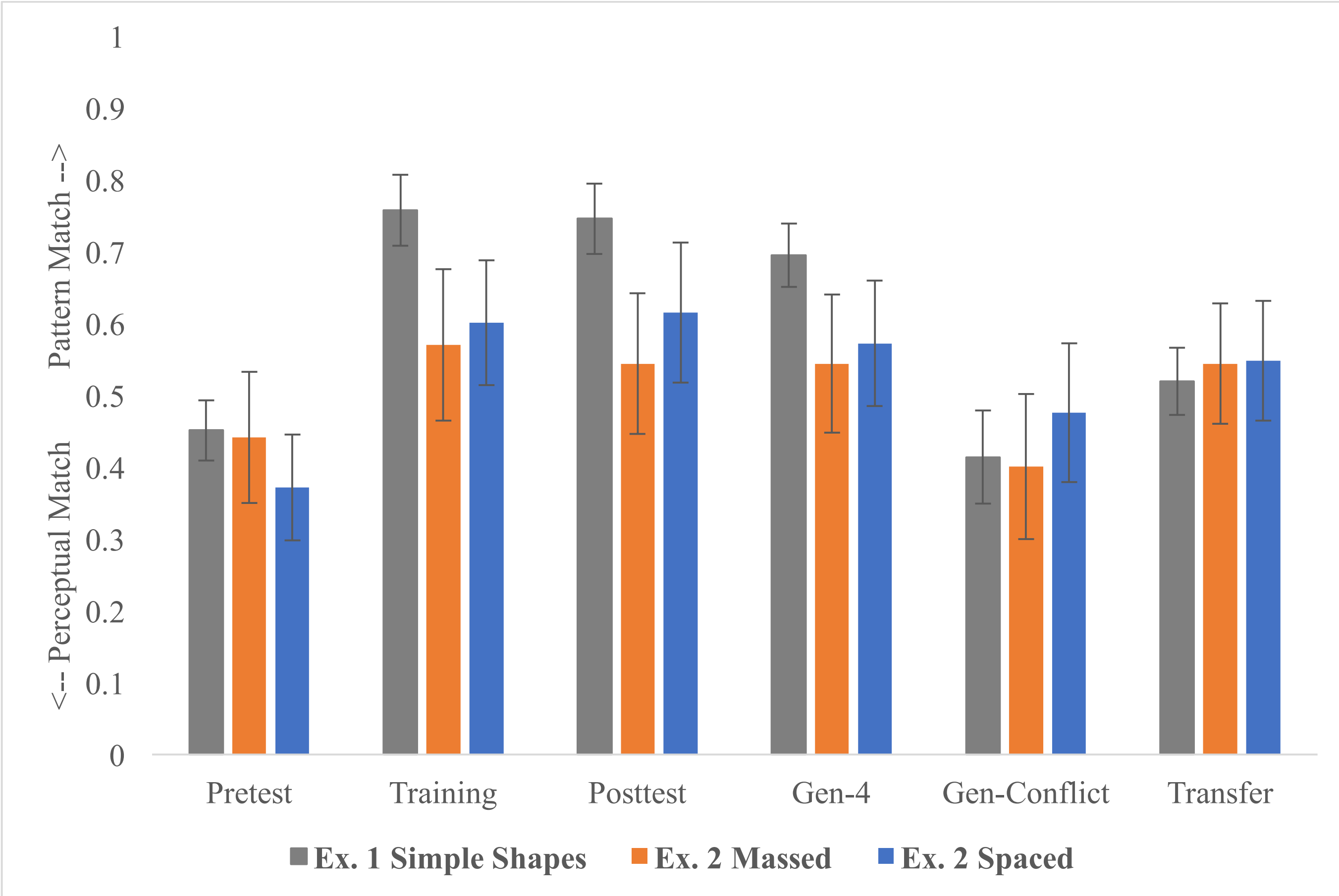


Figure 9. The addition of more training trials to Experiment 2 did not significantly impact trends observed in the Simple Shapes condition of Experiment 1; the trends observed in Experiment 1 were replicated in Experiment 2.

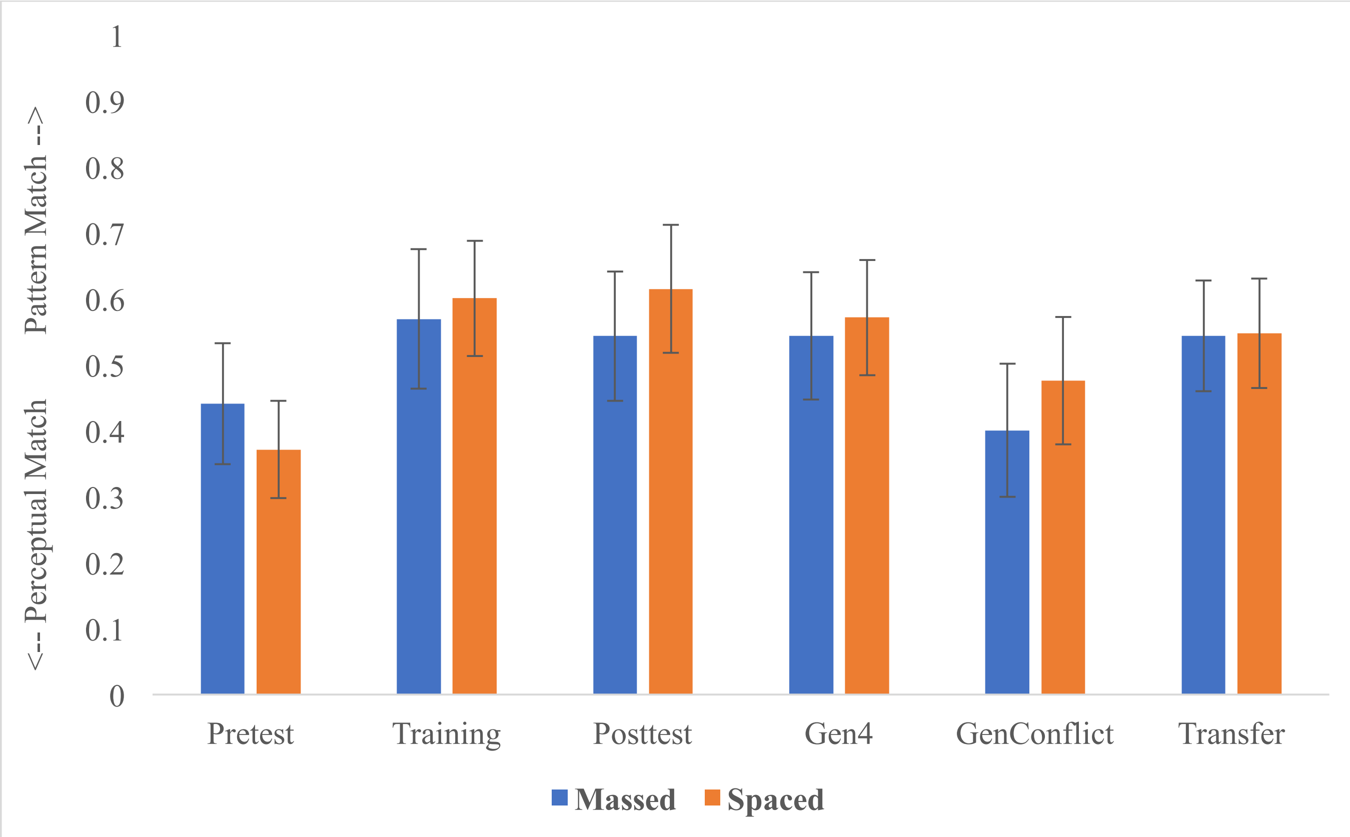


Figure 10. In Experiment 2, children in both conditions showed improved accuracy following training. There was no significant difference in performance between children in the Massed condition vs. children in the Spaced condition as a function of Phase.

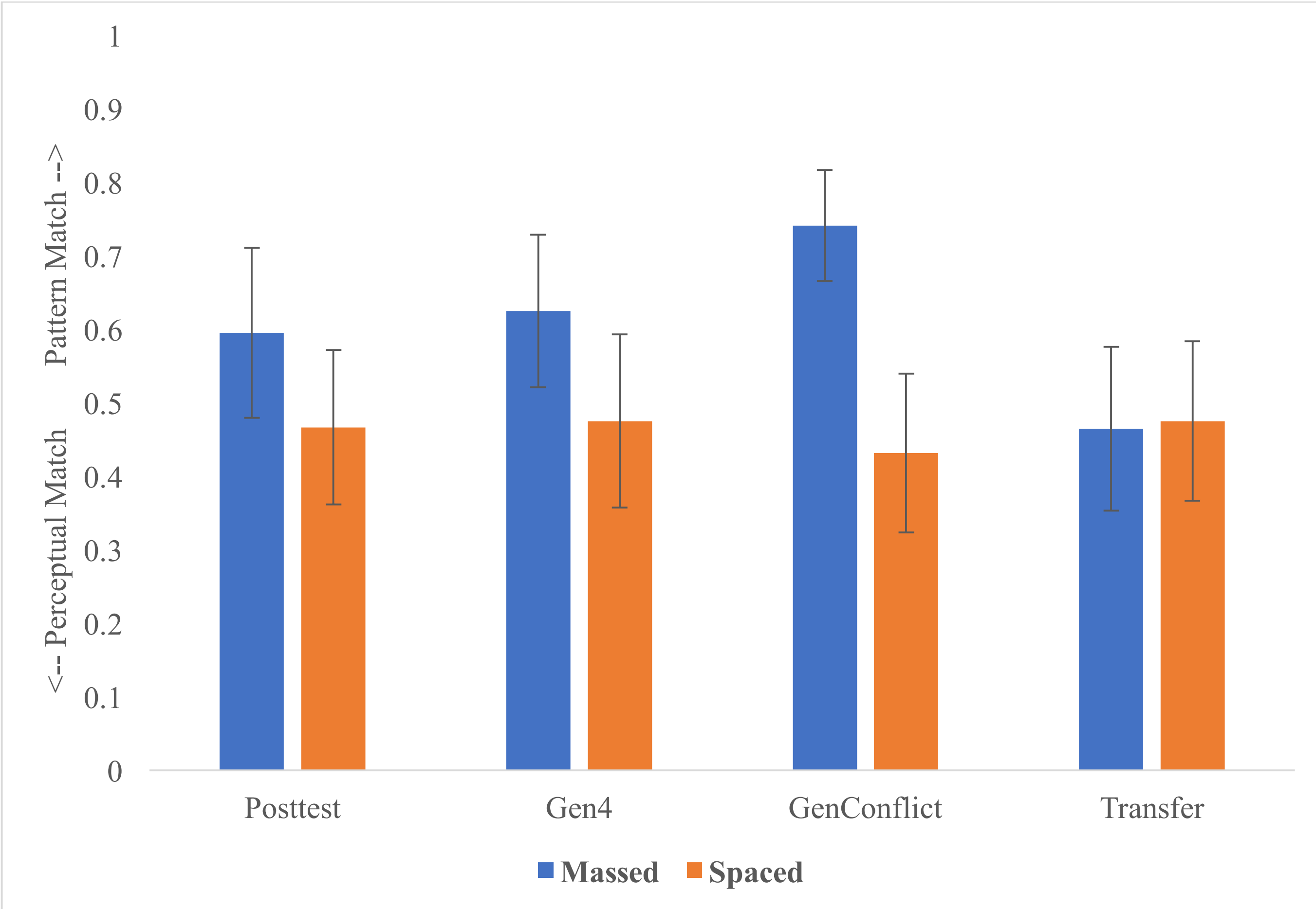
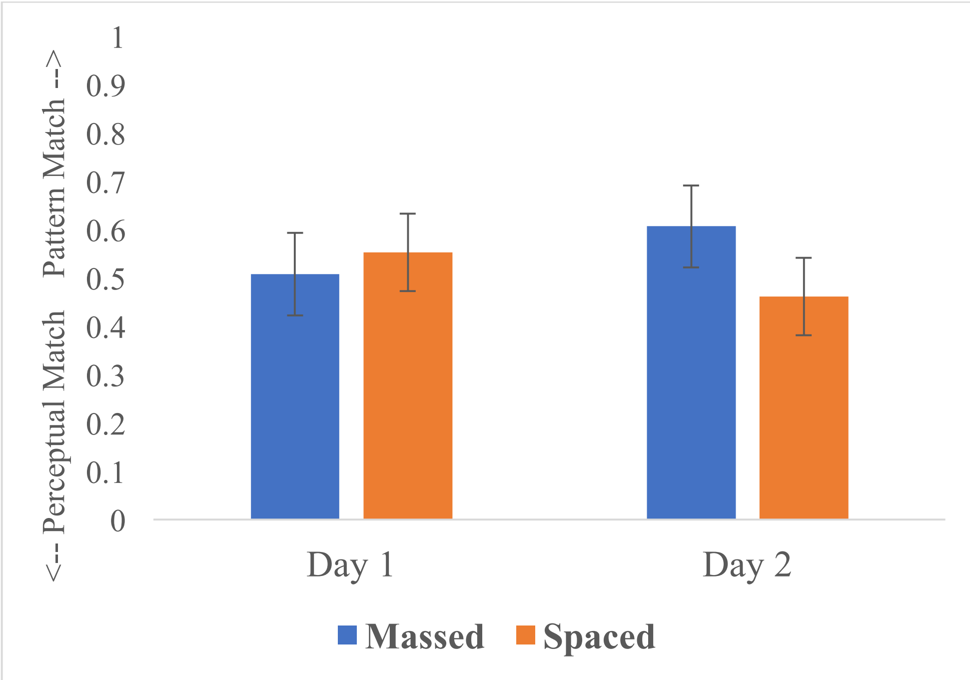
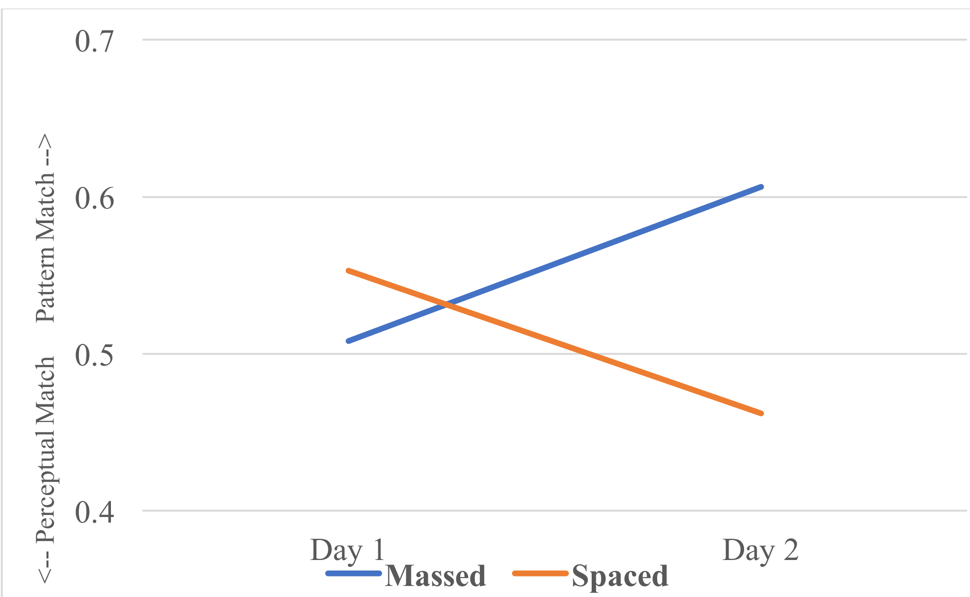


Figure 11. On Day 2 of Experiment 2, there was not a significant difference in performance based on phase. Children had similar accuracy across all phases of the experiment relative to their condition.





Figures 12 and 13. Participants in the Massed condition were more accurate on Day 2 of Experiment 2 compared to their accuracy on Day 1. Participants in the Spaced condition were less accurate on Day 2 compared to their accuracy on Day 1. On Day 2, children in the Massed condition were more accurate than children in the Spaced condition.